Effect of asymmetry on stochastic resonance and stochastic resonance induced by multiplicative noise and by mean-field coupling

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In the paper, we investigate the effect of asymmetry of the potential on stochastic resonance (SR) for a model with an asymmetric bistable potential and driven by additive noise, the signal-to-noise ratio (SNR) for a model with a monostable potential and driven by additive and multiplicative noises, and the SNR for a mean-field coupled model with infinite globally coupling oscillators driven by additive noises. It is shown that for the first model,the asymmetry of the potential can weaken the phenomenon of SR; for the second and third models, a SR induced by multiplicative noise and a different one caused by mean-field coupling are found.

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I. INTRODUCTION

Noise-induced nonequilibrium phenomena in nonlinear systems have recently attracted a great deal of attention in a variety of contexts [1]. In general, these phenomena involve a response of the system that is not only produced or enhanced by the presence of the noise, but that is optimized for certain values of the parameters of the noise. One example is the ''Brownian motors,'' wherein for the Brownian motion in stochastic spatial periodic potentials the spatial asymmetry or noise asymmetry leads to a systematic transport whose magnitude and even direction can be tuned by parameters of the noise [2]. Another is the nonequilibrium transition for the systems with finite or infinite coupled oscillators, which probably is a phase transition (the first order or second order) [3–5] or not [6]. For these systems, the most exciting is that a reentrant second order phase transition has been found for a general spatially extended model by Van den Broeck, Parrondo, and Toral [3]. Afterwards, this phenomenon has been found in a lot of systems with coupled oscillators. A third is the resonant activation [7], here the mean first passage time (MFPT) of a particle driven by (usually white) noise over a fluctuating potential barrier exhibits a minimum as a function of the parameter of the fluctuating potential barrier (usually the flipping rate of the fluctuating potential barrier). A fourth such phenomenon is the phenomenon of stochastic resonance [8–13], the one of interest to us in this paper, wherein the response of a nonlinear system to a signal is enhanced by the presence of noise and maximized for certain values of the noise parameters.

Since the stochastic resonance was proposed by Benzi and co-workers [8] to explain the periodic recurrences of the earth's ice ages, this phenomenon has been extensively investigated from both the theoretical and experimental points of view [9–13].

There have been many theoretical developments of stochastic resonance in conventional bistable systems. McNamara and Wiesenfeld [10] have suggested a master equation for the populations in two stable states. They considered the signal-to-noise ratio (SNR), i.e., the ratio of the peak height in the power spectrum to the noise background as a probe of the stochastic resonance effect. Zhou, Moss, and Jung [11] have suggested the escape time distribution to describe stochastic resonance. Jung and Hänggi [12] described stochastic resonance within the framework of nonstationary stochastic processes without restriction to small driving amplitudes or frequencies, where they presented power spectral densities and signal amplification as measures of stochastic resonance.

However, the above theories for stochastic resonance (SR) deal with the symmetric potential system driven by additive noise, and SR is only induced by additive noise. When one studies a practical problem, it is inevitable to meet the stochastic asymmetric potential system, the system driven by multiplicative noise, and the stochastic coupling oscillators system. So one can ask how the effect of the asymmetry of the potential on SR will be, and whether there will be SR induced by multiplicative noise and one by the coupling among different oscillators. In this paper, we will investigate these problems. Here we will use the SNR to represent the phenomenon of SR. To solve the above problems, the SNR for system with asymmetric potential will be required (see Secs. III, IV, and V). So we will first derive the SNR for system with asymmetric potential in Sec. II. Then in Sec. III, we will study the effect of asymmetry of the potential on SR; and in Secs. IV and V we will show a SR induced by multiplicative noise and a one by mean-field coupling respectively.

II. THE SIGNAL-TO-NOISE RATIO

In this section, we derive the formula of the signal-to-noise ratio for a stochastic system (only driven by additive noise and in dimensionless form) with asymmetric bistable potential (see Fig. 1). In Fig. 1, w_+ and w_- are the transition rates from x_1 to x_2 and from x_2 to x_1 in absence of the external signal. It is clear that $w_+ \neq w_-$. The mean first passage times from x_1 to x_2 and vice versa are respectively [14]

$$T_+(x_1 \to x_2) = \frac{1}{D}\int_{x_1}^{x_2} dy e^{U(y)/D}\int_{\infty}^{y} e^{-U(z)/D}dz, \qquad (1)$$

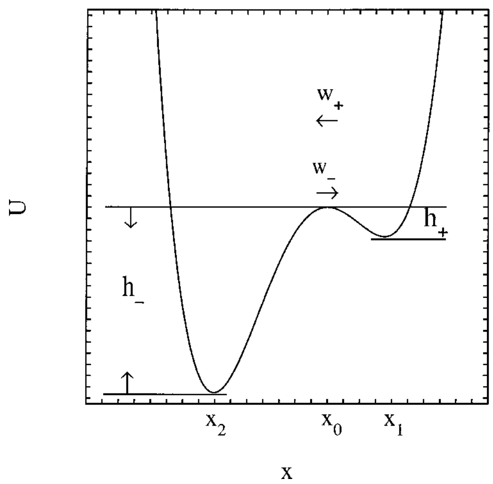

FIG. 1. Asymmetric bistable potential in dimensionless form. The minima are at x_1 and x_2, the maximum at x_0. The transition rates from x_1 to x_2 and vice versa are w_- and w_+.

$$T_-(x_2\rightarrow x_1)=\frac{1}{D}\int_{x_2}^{x_1}dy e^{U(y)/D}\int_{-\infty}^{y}e^{-U(z)/D}dz, \quad (2)$$

where D is the additive noise strength. The transition rates are $w_\pm=1/T_\pm$. When adding the external signal $A\cos\omega t$ we have the transitions rates (in the adiabatic limit)

$$w'_+=D\left\{\int_{x_1}^{x_2}dy\exp\{[U(y)-Ay\cos\omega t]/D\}\times\int_{\infty}^{y}\exp\{-[U(z)-Az\cos\omega t]/D\}dz\right\}^{-1}, \quad (3)$$

$$w'_-=D\left\{\int_{x_2}^{x_1}dy\exp\{[U(y)-Ay\cos\omega t]/D\}\times\int_{-\infty}^{y}\exp\{-[U(z)-Az\cos\omega t]/D\}dz\right\}^{-1}. \quad (4)$$

To the first order in A, from Eqs. (3) and (4) we can obtain

$$w'_\pm \doteq \frac{D}{T_\pm}\left(1+\frac{A\cos\omega t}{D}M^\pm\right), \quad (5)$$

in which $M^\pm=T_1^\pm/T_\pm$, $T_+=\int_{x_1}^{x_2}\int_{\infty}^{y}\exp\{[U(y)-U(z)]/D\}dydz$, $T_1^+=\int_{x_1}^{x_2}\int_{\infty}^{y}(y-z)\exp\{[U(y)-U(z)]/D\}dydz$, $T_-=\int_{x_2}^{x_1}\int_{-\infty}^{y}\exp\{[U(y)-U(z)]/D\}dydz$, and $T_1^-=\int_{x_2}^{x_1}\int_{-\infty}^{y}(y-z)\exp\{[U(y)-U(z)]/D\}dydz$.

We define $n_-=1-n_+=\int_{-\infty}^{\infty}xp(x)dx$, here $p(x)$ is the stationary probability density. Then the governing rate equation is just

$$\frac{dn_+}{dt}=-\frac{dn_-}{dt}=w'_-(t)n_--w'_+(t)n_+ = w'_- - [w'_-(t)+w'_+(t)]n_+. \quad (6)$$

Substituting Eq. (5) into Eq. (6), and solving equation, we can get

$$\begin{aligned}n_+(t|x_0,t_0)=&[X_2A\cos(\omega t_0+\phi_1)-X_1A\cos(\omega t_0-\phi)\\&+AX_3\sin\omega t-X_0+\delta_{x_0x_1}]e^{-Q_0(t-t_0)}\\&+[X_1A\cos(\omega t-\phi)-X_2A\cos(\omega t+\phi_1)\\&-AX_3\sin\omega t+X_0],\end{aligned} \quad (7)$$

where $X_0=D/(T_-Q_0)$, $X_1=M^-/(T_-\sqrt{Q_0^2+\omega^2})$, $X_2=Q_1D/(T_-\omega\sqrt{Q_0^2+\omega^2})$, $X_3=(Q_1/\omega)X_0$, $\phi=tg^{-1}(\omega/Q_0)$, $\phi_1=tg^{-1}(Q_0/\omega)$, $Q_0=D([1/T_-]+[1/T_+])$, and $Q_1=(M^+/T_+)+(M^-/T_-)$. From Eq. (7) the average autocorrelation function can be computed as follows:

$$\begin{aligned}\langle\langle x(t)x(t+\tau)\rangle\rangle_t=&\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\langle x(t)x(t+\tau)\rangle dt\\
=&\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\lim_{t_0\rightarrow-\infty}\langle x(t)x(t+\tau)|x_0,t_0\rangle dt=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\lim_{t_0\rightarrow-\infty}[x_1^2n_+(t+\tau|x_1,t)n_+(t|x_0,t_0)\\
&+x_1x_2n_+(t+\tau|x_2,t)n_-(t|x_0,t_0)+x_1x_2n_-(t+\tau|x_1,t)n_+(t|x_0,t_0)+x_2^2n_-(t+\tau|x_2,t)n_-(t|x_0,t_0)]dt\\
=&x_1^2[X_0^2+A^2B_1\cos\omega\tau+(A^2B_2+B_3)e^{-Q_0\tau}+B_4A^2\sin\omega\tau e^{-Q_0\tau}+B_5A^2\cos\omega\tau e^{-Q_0\tau}]\\
&-2x_1x_2[X_0^2+A^2B_1\cos\omega\tau+(A^2B_2+B_3)e^{-Q_0\tau}+B_4A^2\sin\omega\tau e^{-Q_0\tau}+B_5A^2\cos\omega\tau e^{-Q_0\tau}-2X_0]\\
&+x_2^2[1+X_0^2+A^2B_1\cos\omega\tau+(A^2B_2+B_3)e^{-Q_0\tau}+B_4A^2\sin\omega\tau e^{-Q_0\tau}+B_5A^2\cos\omega\tau e^{-Q_0\tau}-2X_0],\end{aligned} \quad (8)$$

in which

$$M_1=x_1^2X_0^2-2x_1x_2X_0^2+2x_1x_2X_0+x_2^2+x_2^2X_0^2-2x_2^2X_0,$$

$$M_2=x_1^2B_1A^2-2x_1x_2B_1A^2+x_2^2B_1A^2,$$

$$M_3=x_1^2(A^2B_2+B_3)-2x_1x_2(A^2B_2+B_3)+x_2^2(A^2B_2+B_3),$$

$$M_4=x_1^2B_4A^2-2x_1x_2B_4A^2+x_2^2B_4A^2,$$

and

$$M_5=x_1^2B_5A^2-2x_1x_2B_5A^2+x_2^2B_5A^2,$$

with

$$B_1=\frac{1}{2}(X_1^2+X_2^2+X_3^2)-X_1X_2\cos(\phi+\phi_1)-X_1X_3\sin\phi-X_2X_3\sin\phi_1,$$

$$B_2=-\frac{1}{2}(X_1^2+X_2^2)+X_1X_2\cos(\phi+\phi_1)+X_1X_3\sin\phi+\frac{1}{2}X_2X_3\sin\phi_1,$$

$$B_3=X_0(1-X_0),$$

$$B_4=\frac{1}{2}[X_1X_3\cos\phi-X_2X_3\cos\phi_1],$$

and

$$B_5=\frac{1}{2}[X_1X_3\sin\phi+X_2X_3\cos\phi_1-X_3^2].$$

The power spectrum is

$$\begin{aligned}s(\Omega)&=\langle s(\Omega)\rangle_t+\langle s(-\Omega)\rangle_t=2M_1\delta(\Omega)+2\pi M_2(\omega)\delta(\Omega-\omega)+\frac{4M_3(\omega)Q_0}{Q_0^2+\Omega^2}2M_5(\omega)\left(\frac{Q_0}{Q_0^2+(\Omega-\omega)^2}+\frac{Q_0}{Q_0^2+(\Omega+\omega)^2}\right)\\&=G_s^{(0)}\delta(\Omega)+G_s^{(1)}(\omega)\delta(\Omega-\omega)+G_N^{(1)}(\omega,\Omega),\end{aligned} \tag{9}$$

where $\langle s(\Omega)\rangle_t=\int_{-\infty}^{\infty}\langle\langle x(t)x(t+\tau)\rangle\rangle_t e^{-i\Omega\tau}d\tau$.

So the signal-to-noise ratio R_1 can be obtained

$$R_1=\left|\frac{G_s^{(1)}(\omega)}{G_N^{(1)}(\omega,\Omega)}\right|_{\Omega=\omega}=\frac{\pi M_2}{\dfrac{2M_3Q_0}{Q_0^2+\omega^2}+M_5\left(\dfrac{Q_0}{Q_0^2+\omega^2}+\dfrac{1}{Q_0}\right)}. \tag{10}$$

Notice that the spectrum (9) divides naturally into three parts: the zero-frequency output that is a δ function at the zero frequency. [This part is produced by the asymmetry of the bistable potential (If the bistable potential is symmetric, we have $M_1=0$.)]; the signal output that is a δ function at the signal frequency; and the broadband noise outputs that are three Lorentzian bumps centered at $\Omega=0$, $\Omega=-\omega$, and $\Omega=\omega$, respectively.

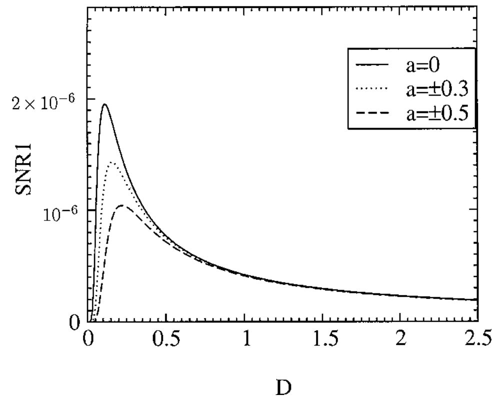

FIG. 2. The signal-to-noise ratio versus the additive noise strength D in dimensionless form with $\omega=0.0005$, $A=0.001$, and $a=0$, ±0.3, and ±0.5 for the model I.

In our calculation, we have used the adiabatic approximation for the transition rate. So our formula (10) is restricted to the condition: the signal frequency is much slower than the inverse value of the relaxation time τ (for double-well system, τ is the time for probability within one well to equilibrate). In addition, the other valid conditions for the formula (10) are: (1) $A/D\ll1$, which is same as that in Ref. [10]; and (2) $h_+\sim h_-$ [see Fig. 1. If $h_+\gg h_-$ (or $h_+\ll h_-$), the formula (10) will be invalid].

III. MODEL I: EFFECT OF THE ASYMMETRY ON SR

Now we consider a special model whose Langevin equation is (in dimensionless form)

$$\dot{x}=-\partial_xU(x)+\eta(t), \tag{11}$$

where $U(x)=\frac{1}{4}x^4-\frac{1}{2}x^2-\frac{1}{3}ax^3$, which is an asymmetric bistable potential, and $\eta(t)$ is noise with zero mean and correlation function $\langle\eta(t)\eta(t')\rangle=2D\delta(t-t')$. When $a=0$, $U(x)$ is symmetric bistable; with the increase of the absolute value of a, $U(x)$ becomes more and more asymmetric. Here we can use $|a|$ to describe the asymmetry of the bistable potential.

If inputting an external periodic signal $F=A\cos\omega t$, we can obtain the phenomenon of SR. In Fig. 2 we plot the signal-to-noise ratio versus the additive noise strength for different values of a ($a=0$, ±0.3, and ±0.5). The figure shows that the asymmetry of the bistable potential can weaken the phenomenon of SR.

If the potential has only one well, no phenomenon of SR appears. If the potential has three or more wells, the phenomenon of SR can emerge. For this case, we study several other examples by using the formula (10). [Now the formula (10) is still applicable, but we should use $w_\pm^{(1)}+w_\pm^{(2)}+\cdots$ to replace $w'_\pm$, where $w_+^{(1)},w_+^{(2)},\ldots$ are the transition rates from the first well to the second well (the particle moves from the right to the left), from the second well to the third well, ..., and $w_-^{(1)}, w_-^{(2)}, \ldots$ are the transition rates for *vice versa*]. Study shows that the asymmetry of the potentials can induce the phenomenon of SR to change. It is determined by the term in the Langevin equation, which induces the asymmetry of the potentials, how the phenomenon of SR changes. For our model (11), it is the effect of the term ax^2 on the system that the phenomenon of SR can be weakened. Because of the variety of the change for the asymmetry of the potentials, here we cannot exclude the case that the asymmetry of the potentials has no effect on the phenomenon of SR. or has more complex effect on this phenomenon.

IV. MODEL II: SR INDUCED BY MULTIPLICATIVE NOISE

In this section, we study a model with monostable potential driven simultaneously by additive and multiplicative noises. The Langevin equation of the model is (in dimensionless form)

$$\dot{x} = -x^3 - 2x^2 + x\xi(t) + \eta(t), \tag{12}$$

in which $\xi(t)$ and $\eta(t)$ are respectively the multiplicative and additive noises with zero means and correlation functions $\langle \xi(t)\eta(t')\rangle = 0$, $\langle \xi(t)\xi(t')\rangle = 2D_1\delta(t-t')$, and $\langle \eta(t)\eta(t')\rangle = 2D\delta(t-t')$.

The Fokker-Planck equation of Eq. (12) is

$$\partial_t P(x,t) = -\partial_x(-x^3 - 2x^2)P(x,t) + D_1\partial_x x \partial_x x P(x,t) + D\partial_x^2 P(x,t). \tag{13}$$

The stationary solution of Eq. (13) is

$$P_s(x) = Me^{-U_{eff}(x)/D}, \tag{14}$$

where $U_{eff}(x) = \int^x \{(x^3 + 2x^2 + D_1x)/[(D_1/D)x^2 + 1]\}dx$, which is the stationary effective potential of the system, and M is the normalization constant.

If we only consider the stationary state, Eq. (12) can be written as

$$\dot{x} = -\partial_x U_{eff}(x) + \eta(t). \tag{15}$$

If inputting an external signal $A\cos\omega t$ to Eq. (12), the corresponding equation for Eq. (15) becomes

$$\dot{x} = -\partial_x U_{eff}(x) + \eta(t) + \frac{A\cos\omega t}{\frac{D_1}{D}x^2 + 1}. \tag{16}$$

In Eq. (16) the external signal depends on x, so the formula (10) of the SNR is not applicable to it. But if we use

$$\overline{T_1^+} = \int_{x_1}^{x_2}\int_{\infty}^{-\infty}\sqrt{\frac{D}{D_1}}\left[tg^{-1}\left(\sqrt{\frac{D_1}{D}}y\right) - tg^{-1}\left(\sqrt{\frac{D_1}{D}}z\right)\right] \times \exp\{[U_{eff}(y) - U_{eff}(z)]/D\}dydz,$$

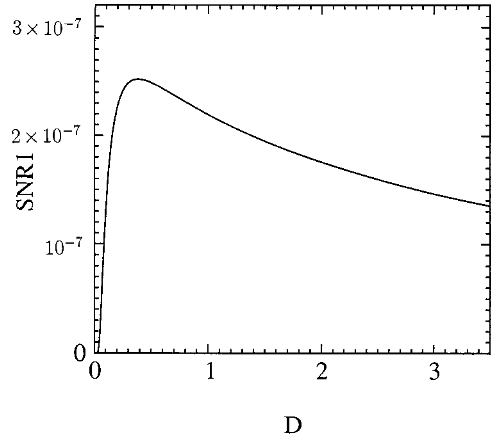

FIG. 3. The signal-to-noise ratio versus the additive noise strength D in dimensionless form with the multiplicative noise strength $D_1 = 0.7$, $A = 0.001$, and $\omega = 0.0005$ for the model II.

and

$$\overline{T_1^+} = \int_{x_2}^{x_1}\int_{-\infty}^{\infty}\sqrt{\frac{D}{D_1}}\left[tg^{-1}\left(\sqrt{\frac{D_1}{D}}y\right) - tg^{-1}\left(\sqrt{\frac{D_1}{D}}z\right)\right] \times \exp\{[U_{eff}(y) - U_{eff}(z)]/D\}dydz$$

to replace T_1^+ and T_1^- respectively in Eqs. (5), (7)–(10), the corresponding formula will be applicable to Eq. (16). The SNR versus the additive noise strength D is plotted in Fig. 3 with $D_1 = 0.7$ [in order to make our calculation satisfy the valid condition $h_+ \sim h_-$ of the effective potential for the formula (10), in this figure we set the multiplicative noise strength as $D_1 = 0.7$]. The figure shows that there is the phenomenon of stochastic resonance. But this phenomenon of stochastic resonance is different from the one for model I, since it is a different one caused by the multiplicative noise. In the absence of the multiplicative noise, there is no phenomenon of stochastic resonance. In addition, we find that it is not for all the cases of Eq. (12) there is the phenomenon of stochastic resonance. This can be observed from the structure of the effective potential $U_{eff}(x)$ of Eq. (12), which indicates that only when $0 < D_1 < 1$ this phenomenon can appear.

Below we consider the cases when the multiplicative noise $\xi(t)$ is other types of noise, such as *O-U* noise, dichotomous noise, and Poisson noise.

If $\xi(t)$ is O-U noise with zero mean and correlation function $\langle \xi(t)\xi(t')\rangle = (D_1/\tau)\exp(-|t-t'|/\tau)$, one can obtain the following approximate Fokker-Planck equation for small correlation time τ applying the UCNA to the Eq. (12) [15]

$$\partial_t P(x,t) = -\partial_x A(x)P(x,t) + \partial_x^2 B(x)P(x,t),$$

where

$$A(x) = \frac{-x^3 - 2x^2}{1 + 2\tau(x^2 + x)} + \frac{2D_1x}{[1 + 2\tau(x^2 + x)]^2} - \frac{2\tau(2x+1)(D_1x^2 + D)}{[1 + 2\tau(x^2 + x)]^3},$$

$$B(x)=\frac{D_1x^2+D}{[1+2\tau(x^2+x)]^2}.$$

The stationary effective potential is

$$U_{eff}(x)=D\int^x[\partial_x B(x)-A(x)]/B(x)dx.$$

Then, using the method proposed by us in this section we can calculate the SNR when inputting an external periodic signal $F=A\cos\omega t$. Calculation indicates that there is the phenomenon of SR induced by the multiplicative *O-U* noise. [In addition, we still note that the correlation time τ of the multiplicative *O-U* noise for model (12) can enhance the phenomenon of SR].

If $\xi(t)$ is dichotomous noise or Poisson noise, because the values of noise are allowed to take on two discrete values or more discrete values, the phenomenon of SR can appear even if the system is linear [16]. Now the phenomenon of SR is also induced by the multiplicative noise.

V. MODEL III: SR INDUCED BY MEAN-FIELD COUPLING

In this section, we consider a system with infinite globally coupled oscillators. The Langevin equations of the oscillators are (in dimensionless form)

$$\dot{x}_i=-x_i^3+x_i^2+\frac{3}{2}x_is+\eta_i(t),\quad i=1,2,3,\ldots, \tag{17}$$

where s is the mean field and $s=\lim_{N\to\infty}(1/N)\Sigma_{i=1}^N x_i$, and $\eta_i(t)$ are the noise that is similar to the one in Eq. (11). In Eq. (17) the coupling parameter is taken as 3/2, which is for the sake of making the potential of Eq. (17) satisfy the valid condition $h_+\sim h_-$ for the formula (10).

In the case of $N\to\infty$, all the oscillators have an identical evolution given by the nonlinear stochastic equation

$$\dot{x}=-x^3+x^2+\frac{3}{2}xs+\eta(t), \tag{18}$$

in which $s(t)=\langle x(t)\rangle$, which represents the time-dependent order parameter.

The Stratonovich interpretation for Eq. (18) yields the Fokker-Planck Equation

$$\partial_t P(x,s,t)=-\partial_x\left(-x^3+x^2+\frac{3}{2}sx\right)P(x,s,t)+D\partial_x^2P(x,s,t). \tag{19}$$

Under the natural boundary condition, the stationary solution of Eq. (19) is

$$P(x,s)=M_0e^{-U(x,s)/D}, \tag{20}$$

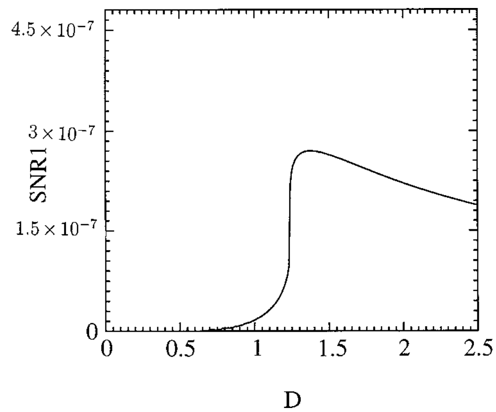

FIG. 4. The signal-to-noise ratio versus the additive noise strength D in dimensionless form with $\omega=0.0005$ and $A=0.001$ for the model III.

where $U(x,s)=\frac{1}{4}x^4-\frac{1}{3}x^3-\frac{3}{4}sx^2$, and M_0 is a normalization constant.

In the limit of $N\to\infty$, the self-consistent Weiss mean-field approach of Desai and Zwanzig is valid [17] and the Weiss mean field has to comply with the condition

$$s=\int_{-\infty}^{\infty}xP(x,s)dx, \tag{21}$$

this is a self-consistency equation whose solution yields the dependence of s with the system parameters.

We first turn to a more detailed analysis of Eq. (21). The trivial solution $s=0$ does not exist. The Eq. (21) only has nonzero solution $s\neq 0$. Thus for the model (17), there is no nonequilibrium transition between the state $s=0$ and the state $s\neq 0$.

When adding an external periodic signal $A\cos\omega t$ we can calculate the SNR of this model from the formula (10). In Fig. 4, the SNR versus D is plotted. From the figure we can find that there is the phenomenon of stochastic resonance for the SNR versus the additive noise strength. If we do not consider the coupling, this phenomenon is absent [the potential function of Eq. (17) will be monostable]. So the phenomenon of stochastic resonance appearing here is a different one, which is caused by the mean-field coupling.

We have noted that in Ref. [18] Zaikin, Kurths, and Schimansky-Geier investigated a mean-field coupled model (a nonlinear lattice of coupled overdamped oscillators) and found stochastic resonance for the symmetric bistable mean field in the presence of small periodic signal. They called this effect doubly stochastic resonance. For our model (17), there is no such phenomenon (the mean field is not bistable). Moreover, in order to determine if all the mean-field coupled models with asymmetric two-well (or multiwell) potential have the phenomenon of stochastic resonance, we have made a lot of numerical calculations for different models. We find that for some models there is the phenomenon of stochastic resonance, but for others there is not this phenomenon even if the potential is two well (or multiwell). Below we give a example for a mean-field coupled model with asymmetric

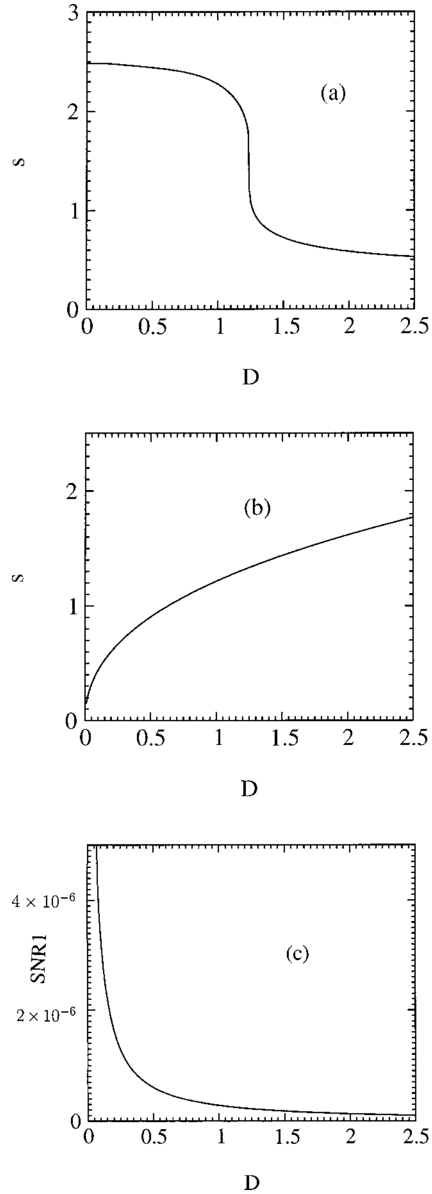

FIG. 5. The mean fields versus the additive noise strengths in dimensionless form for the model III (a), the model (22) (b), and the signal-to-noise ratio versus the additive noise strength in dimensionless form with $\omega = 0.0005$ and $A = 0.001$ for the model (22) (c).

two-well potential but without the phenomenon of stochastic resonance. The Langevin equations of the model are

$$\dot{x}_i = -x_i^3 + (x_i^2 + x_i)s + \eta_i(t), \quad (i = 1,2,3,\ldots), \tag{22}$$

where s and $\eta_i(t)$ are same as the ones in Eq. (17). In Figs. 5(a) and 5(b), we plot the mean fields versus the additive noise strengths for the models (17) and (22) respectively. From the figures we can find that for the model (17) the mean field decreases progressively with the increase of the additive noise strength; while for the model (22) the mean field increases successively with increasing the additive noise strength. It is just because of the difference of the change of the mean fields for different models with varying the additive noise strength that causes for the model (17) there is the phenomenon of stochastic resonance, but for the model (22) there is not. To illustrate there is not the phenomenon of stochastic resonance for the model (22), in Fig. 5(c) we depict the SNR as a function of D.

VI. CONCLUSION AND DISCUSSION

In conclusion, we have studied the effect of asymmetry of the potential on SR for a model with an asymmetric bistable potential and driven by additive noise, the SNR for a model with a monostable potential and driven by additive and multiplicative noises, and the SNR for a mean-field coupled model with infinite globally coupling oscillators driven by additive noises. For the first model, we find that the asymmetry of the potential can weaken the phenomenon of SR; for the second and third models, we find a phenomenon of SR induced by multiplicative noise different a new one by the mean-field coupling.

In Sec. II, we derive a formula for the signal-to-noise ratio by using the adiabatic approximation for system with asymmetric bistable potential. Although this formula is derived for the system with asymmetric bistable potential, it is applicable to the systems with asymmetric multiwell (three or more wells) potential. In addition, in our calculation for this formula we only approximate $w'_{\pm}$ to the first order in A. If we consider the high order, the power spectrum formula will contain the terms of high frequencies. For example, if we approximate $w'_{\pm}$ to the nth order in A, the power spectrum will become

$$s(\Omega) = G_s^{(0)}\delta(\Omega) + \sum_{i=1}^{n} [G_s^{(i)}(i\omega)\delta(\Omega - i\omega) + G_N^{(i)}(i\omega,\Omega)]. \tag{23}$$

Then the signal-to-noise ratio R_i at the frequencies $i\omega$ is

$$R_i = \left| \frac{G_s^{(i)}(i\omega)}{\sum_{i=0}^{n} G_N^{(i)}(i\omega,\Omega)} \right|_{\Omega = i\omega}, \quad i = 1,2,3,\ldots,n. \tag{24}$$

In Eq. (24) owing to $A/D \ll 1$, we can get $R_1 \gg R_2 \gg \cdots \gg R_n$.

In Sec. IV, we propose a method to calculate the SNR for a system simultaneously driven by additive and multiplicative noises. We note that in Ref. [19], Jia, Yu, and Li investigated stochastic resonance for a symmetric bistable system with additive and multiplicative noises and calculated the SNR for the system. Our model II is different from the one studied in Ref. [19] even though they are both driven by additive and multiplicative noises. The former has a bistable potential, while the latter has a monostable one. In addition, in our calculation we have not used the approximate Kramers time, but in the calculation made by Jia, Yu, and Li they have.

Finally, it must been mentioned that for the formula (10) the adiabatic approximation is valid only when $\omega \ll U''(x_1)$ and $U''(x_2)$ (in dimensionless form). In our calculation, in order to satisfy this condition we take a very small value of ω ($\omega = 0.0005$).

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